How expensive is fault-tolerant random circuit sampling?

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Quantum

Background - Sampling random quantum circuits

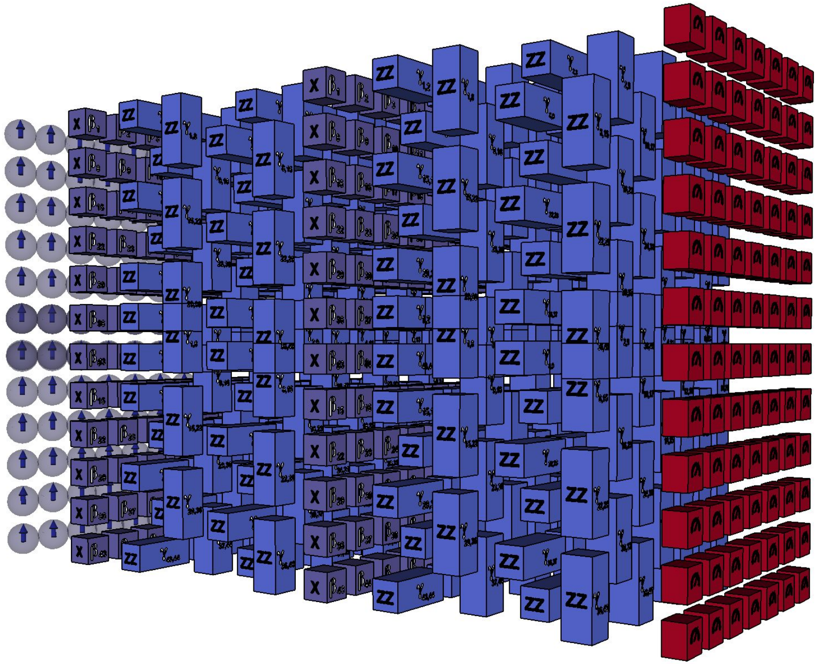
Is it a good problem for quantum advantage?

PROS:

- Trivial for quantum computers.
- Hard for classical computers.
- NISQ compatible.

CONS:

- Not directly useful.
- Expensive to verify.



Goals of this talk

Surface:

Estimate the cost of random circuit sampling using fault tolerant techniques (instead of NISQ techniques).

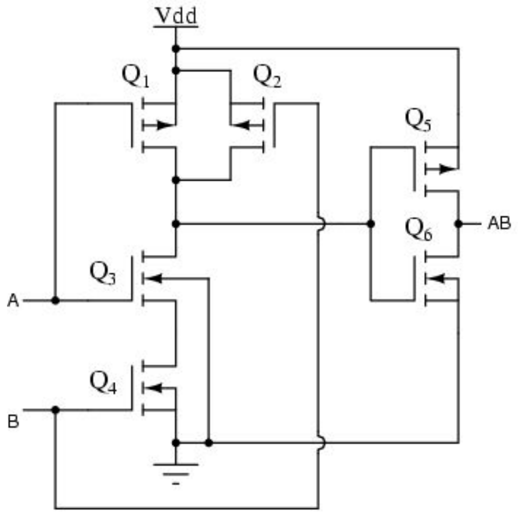
*Underneath***:**

Understand the minimum cost of classically intractable fault tolerant tasks.

Define an intermediate goal that experimentalists can target.

Set a baseline that can be used to track improvements to error correction.

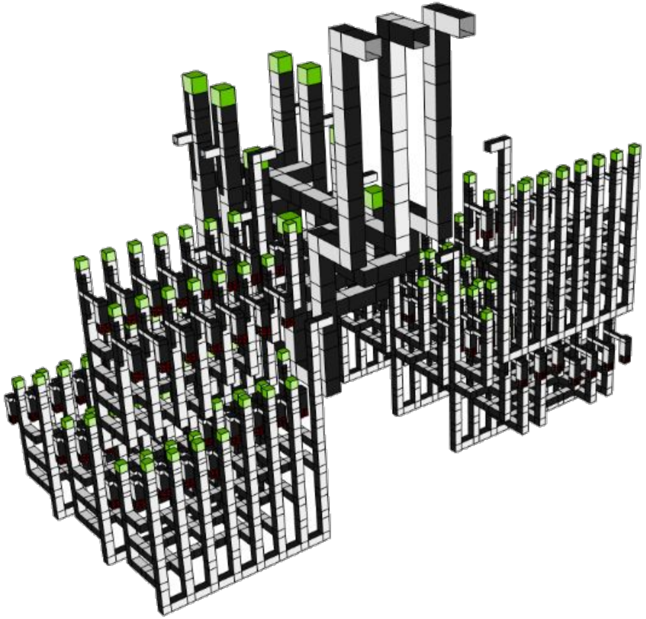
Quantum fault tolerance is expensive



Classical AND gate
(with incredibly low error)

<0.000000001

transistor seconds

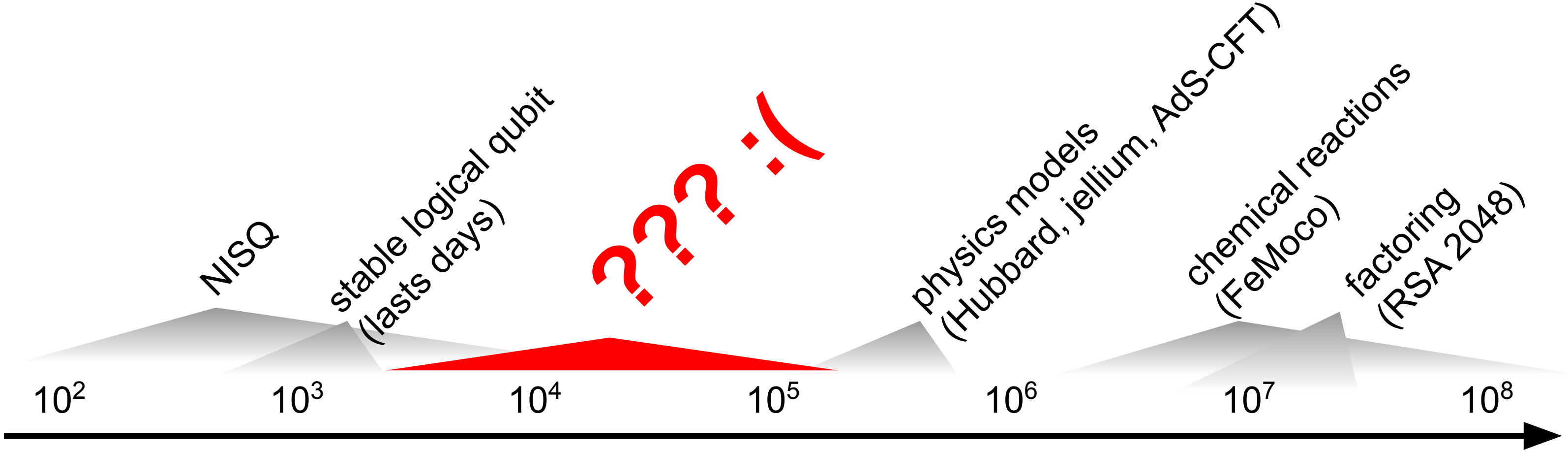


Fault tolerant quantum AND gate
(with error rate good enough for Shor)

20

qubit seconds

There's a chasm on the way to fault tolerance



Projected physical qubit counts for classically intractable fault tolerant tasks
(assuming 5dB error suppression per code distance)
(using current techniques)

The cost of fault tolerance is not stagnant

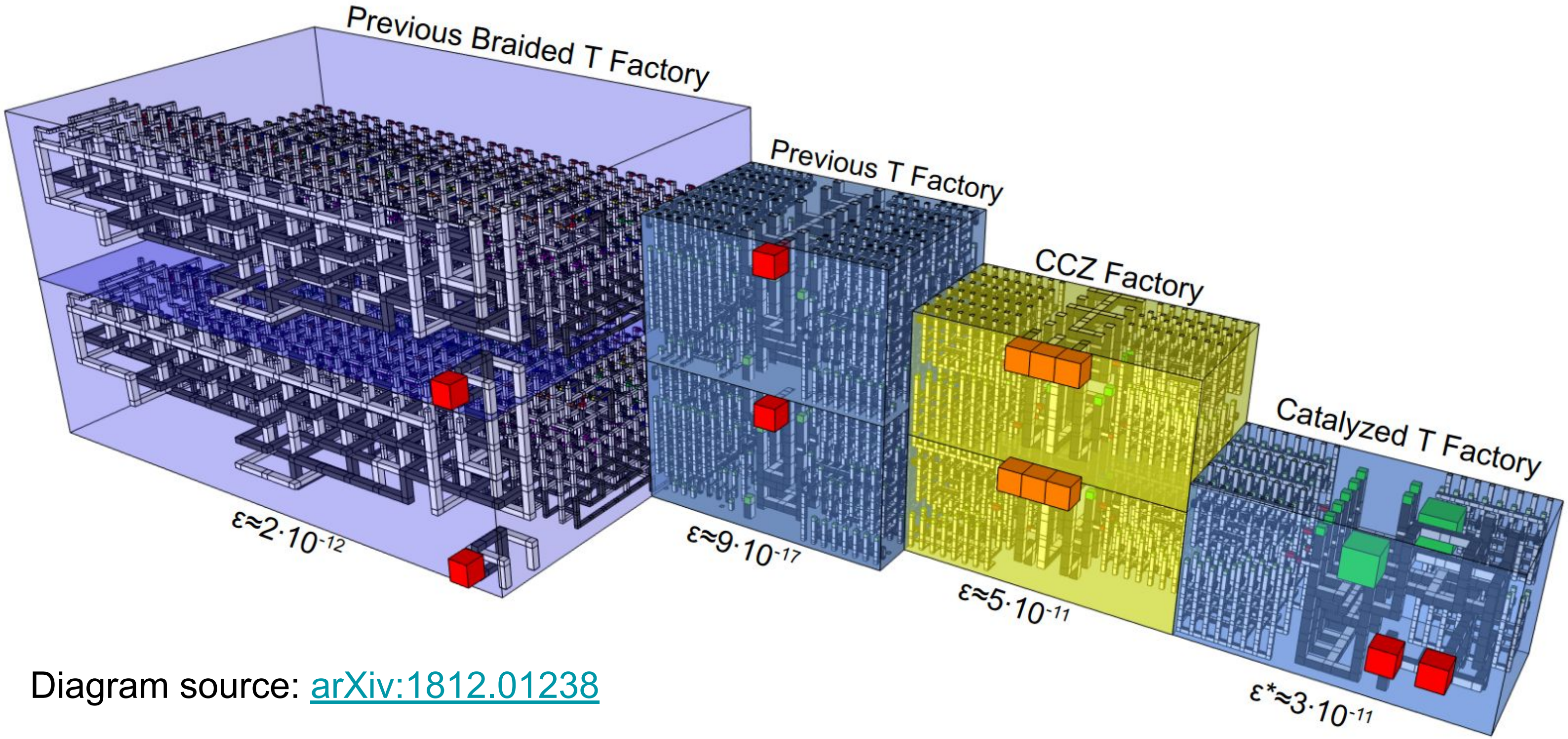


Diagram source: [arXiv:1812.01238](https://arxiv.org/abs/1812.01238)

Recent ideas for further significant improvement explored in [arXiv:1905.06903](https://arxiv.org/abs/1905.06903)

Goals of this talk (reiterated)

Understand the minimum cost of classically intractable fault tolerant tasks.

Define an intermediate goal that experimentalists can target.

Set a baseline that can be used to track improvements to error correction.

Building Blocks

Hardware requirements

Quality: 5 dB of logical error suppression per code distance

code_distance += 2 ⇒ logical_error_rate /= 10

e.g.

$\approx 10^{-3}$ two qubit gate error

$\approx 10^{-2}$ measurement gate error

Speed: 1 microsecond surface code cycle time

e.g.

1 Measurement + 8 CZ + 2 Hadamard in sequence

Distance **d=13** surface code logical qubit patch

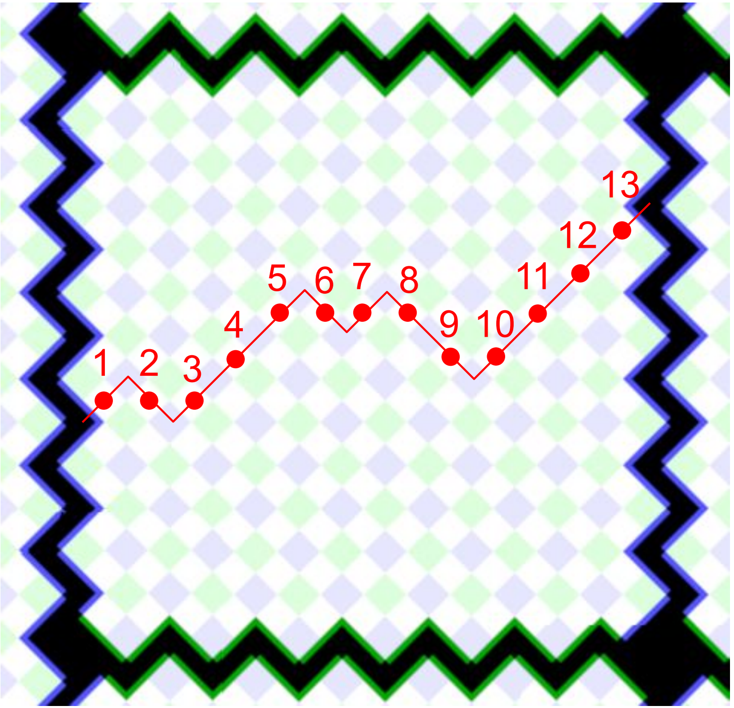
Z measure qubit

X measure qubit

data qubit

disabled qubit

example min-length
Z error chain



physical qubit count

$= 2(d+1)^2$

$= 392$

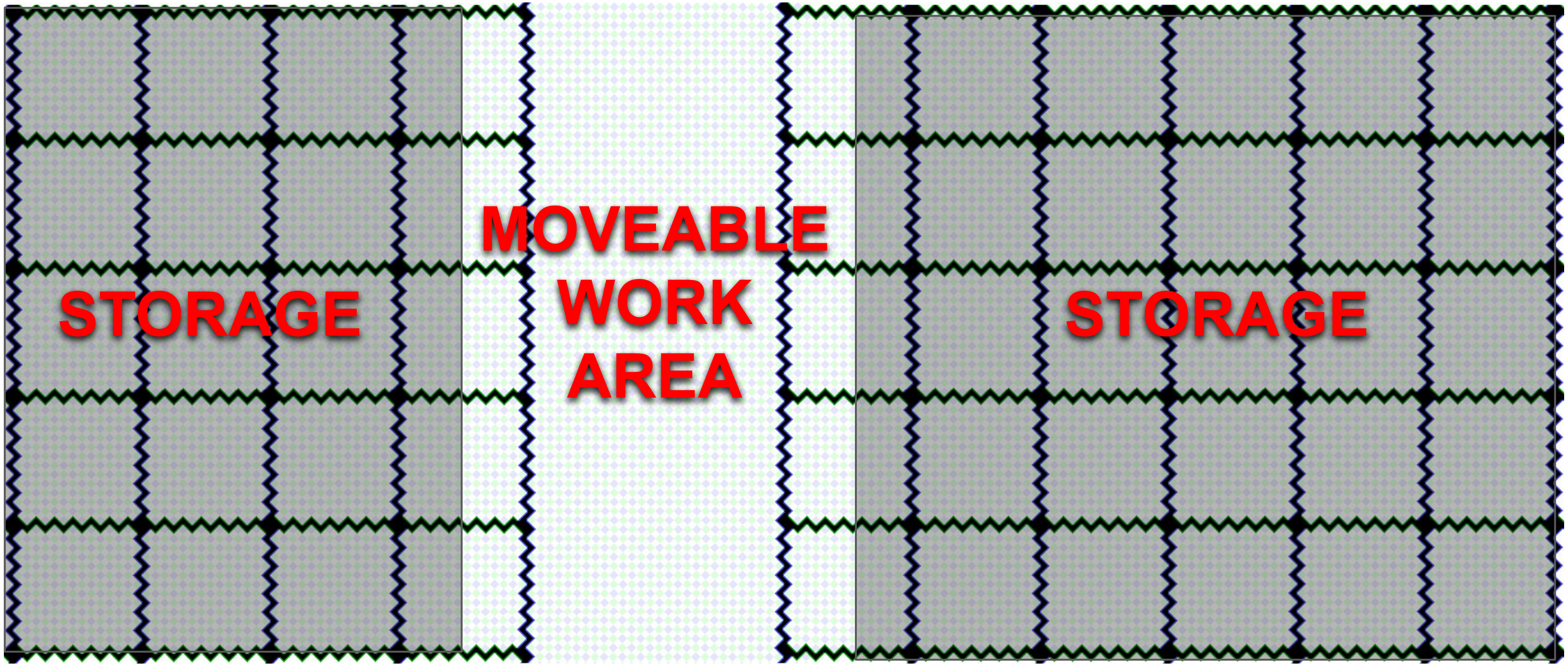
logical error rate per cycle

$\approx -5*(d+3)$ dB

$\approx 10^{-8}$

No-error half-life of ≈1 minute

Layout: 12x5 board with 2 columns unoccupied



Stores 50 d=13 logical qubits (60 including work area)
Total physical qubit count = $12*5 * 2*(13+1)^2 \approx 23.5K$
No-logical-errors-at-all-anywhere half-life of ≈1 second

Low-footprint Hadamard

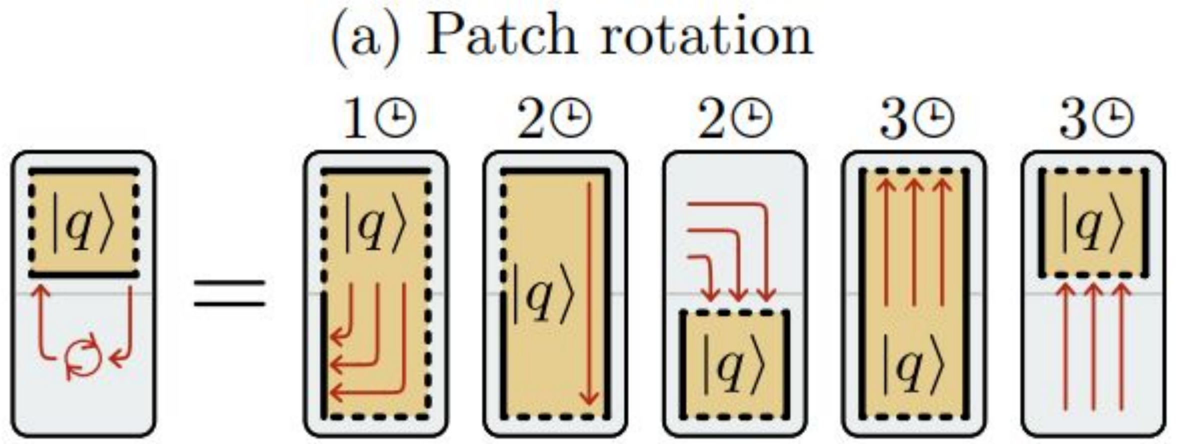
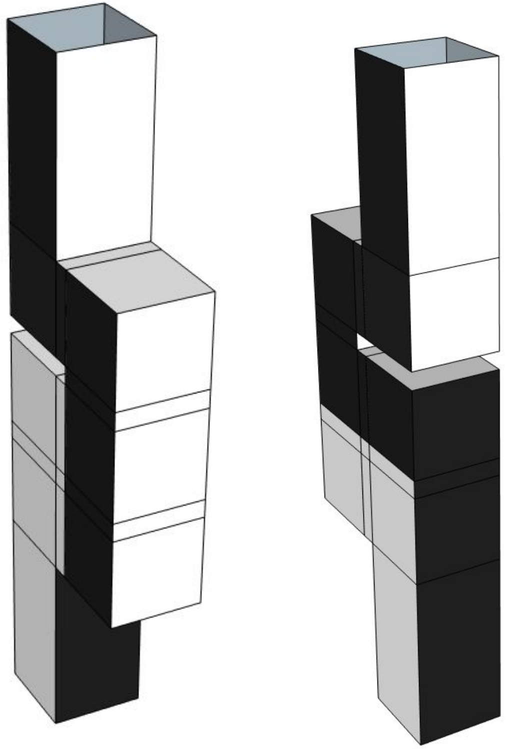
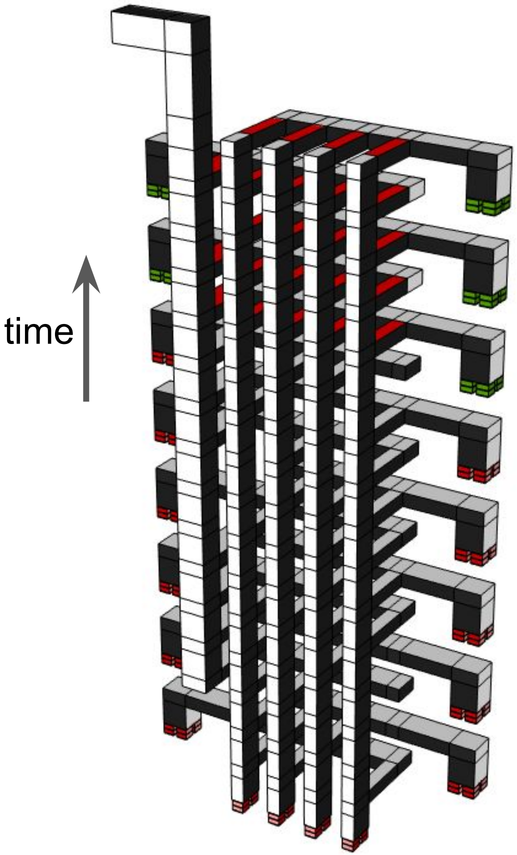


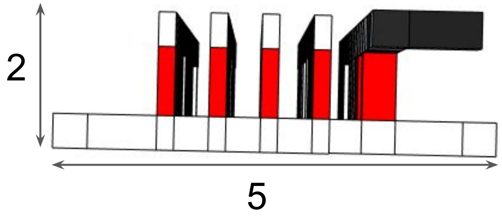
Figure source: [arXiv:1808.02892](https://arxiv.org/abs/1808.02892)



Low footprint T state distillation



Top down view



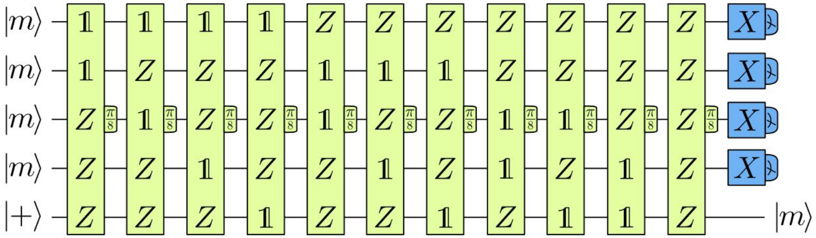
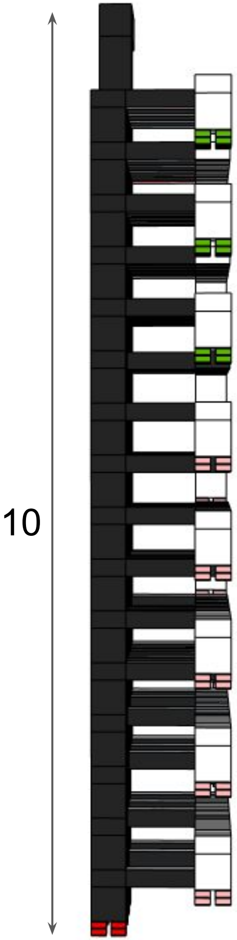


Figure 15: 15-to-1 distillation circuit that uses 5 qubits and 11 $\pi/8$ rotations.

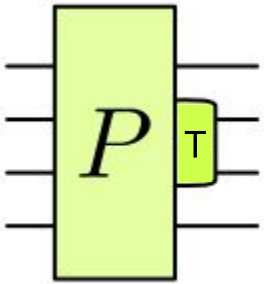
Figure source: arXiv:1808.02892

Independently reported in arXiv:1905.06903

Rear view



Generalized T gate: phasing *products* of Paulis



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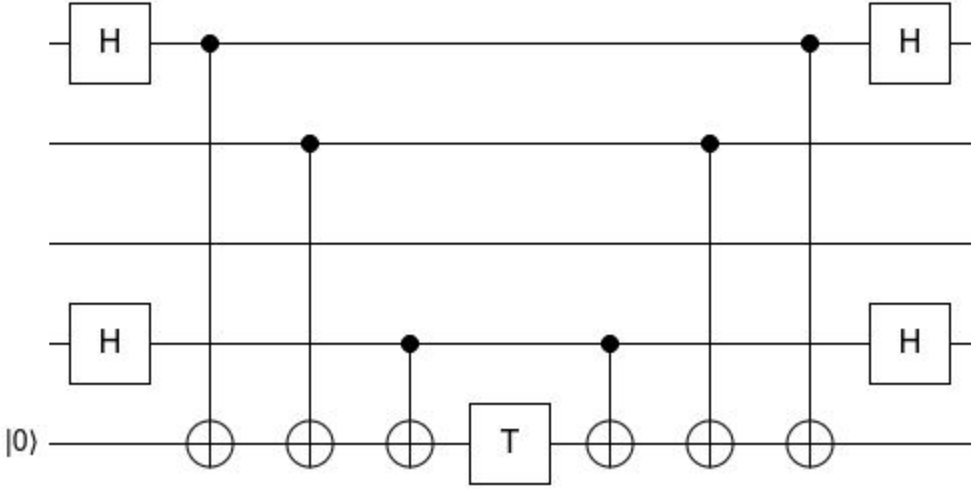
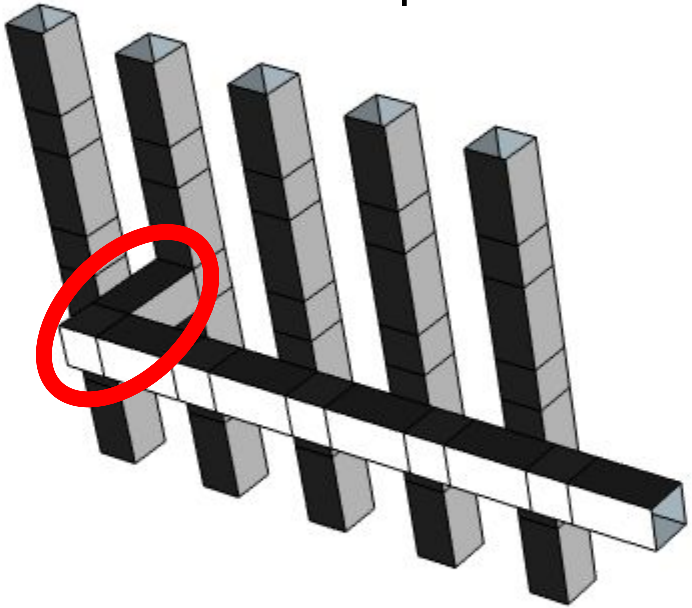


-1 eigenstates of P phased by 45 degrees
+1 eigenstates of P phased by 0 degrees

(circuit is for $P = X_0 Z_1 X_3$)

Generalized T gate has an efficient spacetime layout

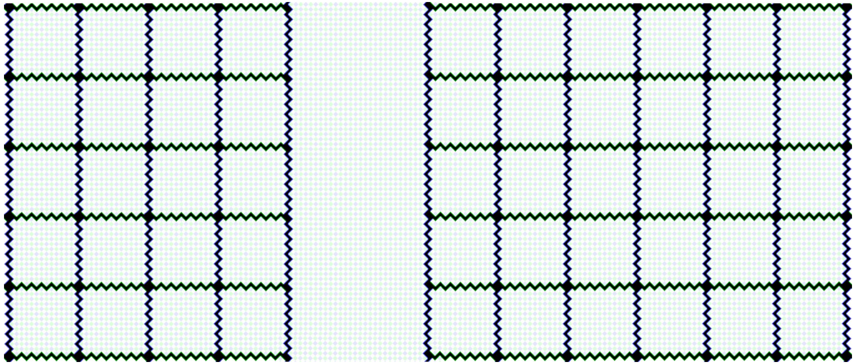
Apply T to Z_1

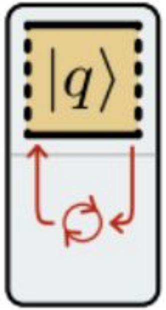


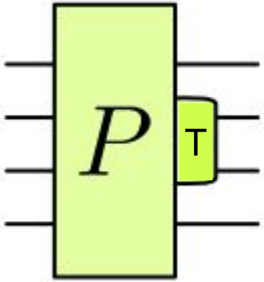
Apply T to $Z_0Z_1Z_3$

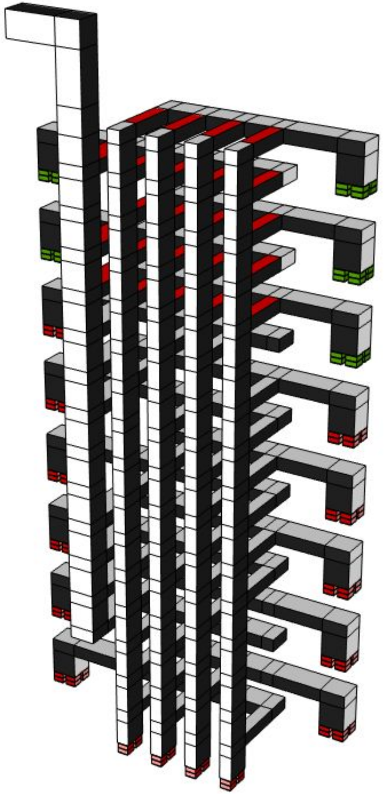
Access X observables via
Hadamard operations

Building blocks (reiterated)



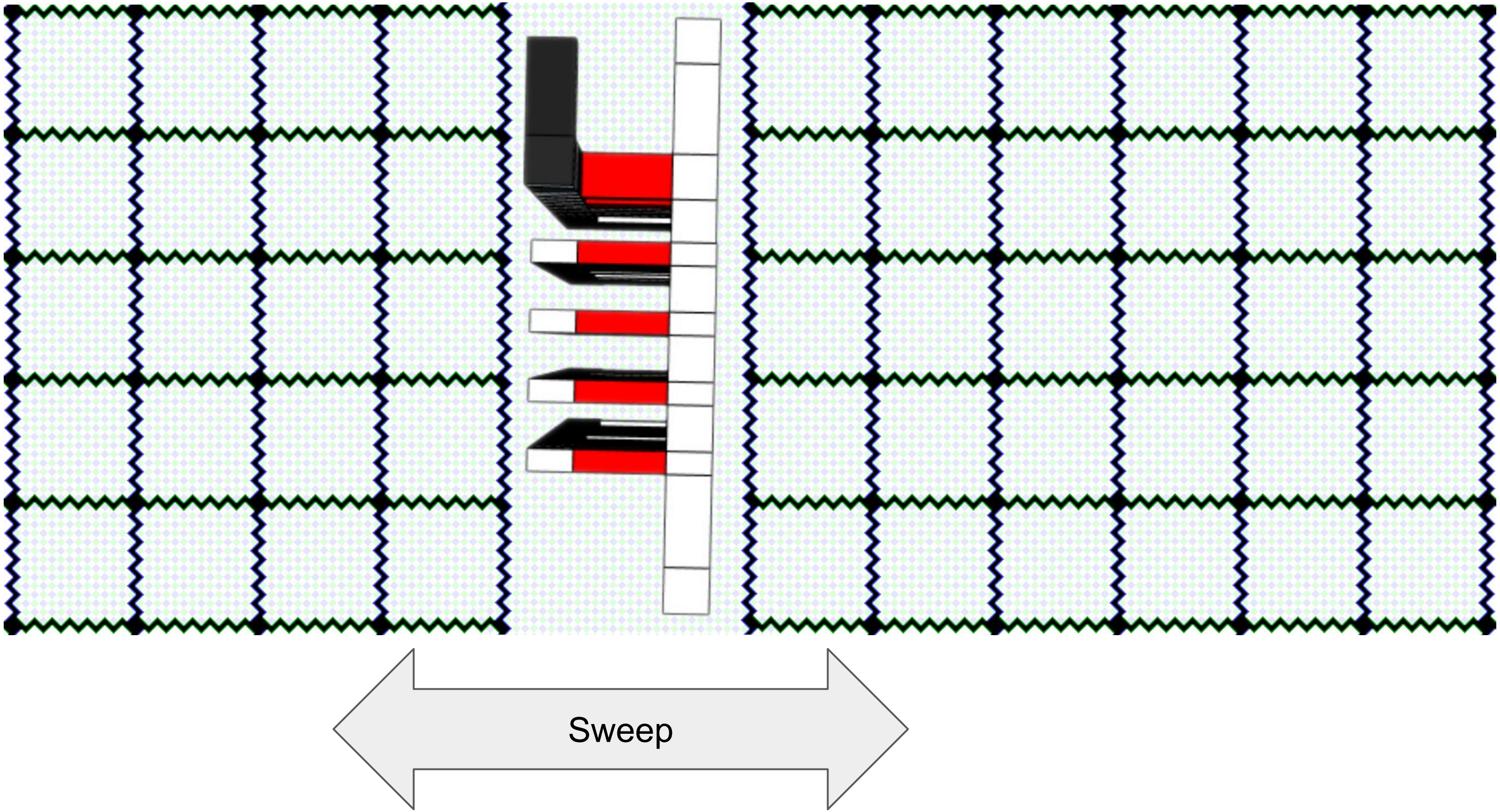




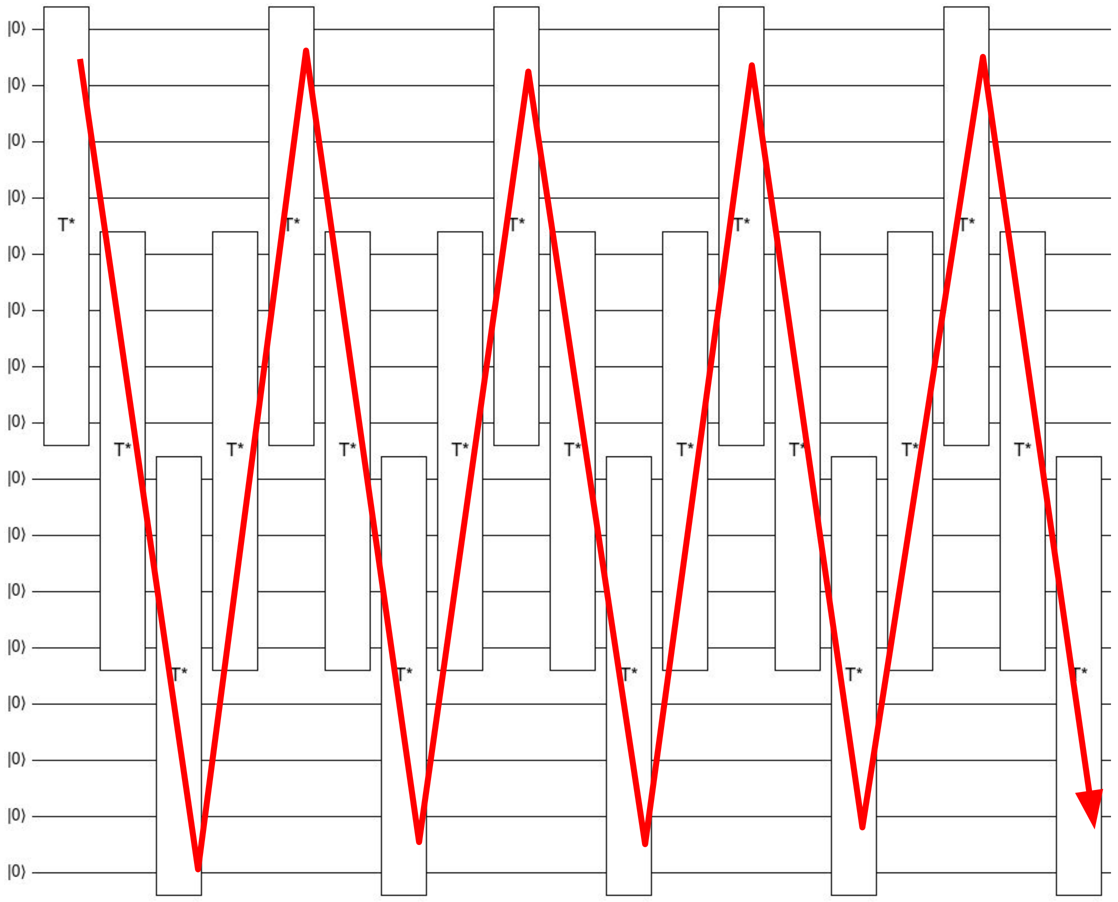


Algorithm and Cost

Mix up state using sweeps of generalized Ts



Mix up state using sweeps of generalized Ts



Sweep operating area back and forth for as long as possible while applying generalized Ts to adjacent qubits

Pick random non-commuting X/Z Pauli products to phase

Back of the envelope generalized T gate rate

+ 3d cycles to Hadamard Xs into Zs

+ 10d cycles to distill a T state

+ 1d cycles to measure the P*T observable

+ 2d cycles to apply S gate fixup

+ 1d cycles to shift operating area

= 17d cycles total (with d=13)

≈ 17*13us = 221 microseconds per generalized T **≈ 4kHz**

Back of the envelope achievable gate count

4kHz generalized T

1 Hz board decay

A thousand generalized Ts would achieve

sufficient signal: O(10%) chance of error, 4 samples per second

sufficient mixing: O(100) sweeps of the operating area

Summary

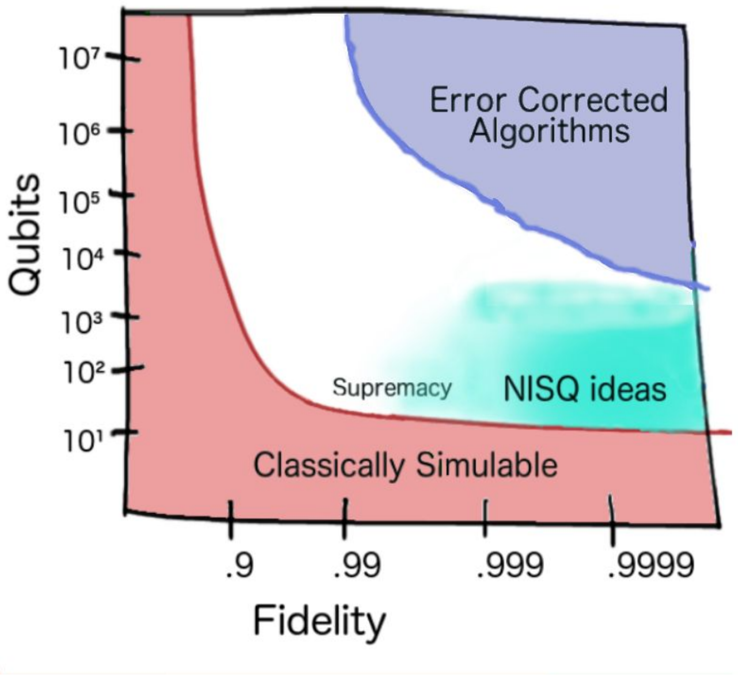
Current techniques for fault tolerant quantum computation are expensive enough that there is a gap from 10K-100K physical qubits where it's difficult to do anything new and interesting.

Using current error correction techniques, and plausible hardware assumptions, fault-tolerant classically-intractable sampling can be done in 1/4 of a second with 25K physical qubits.

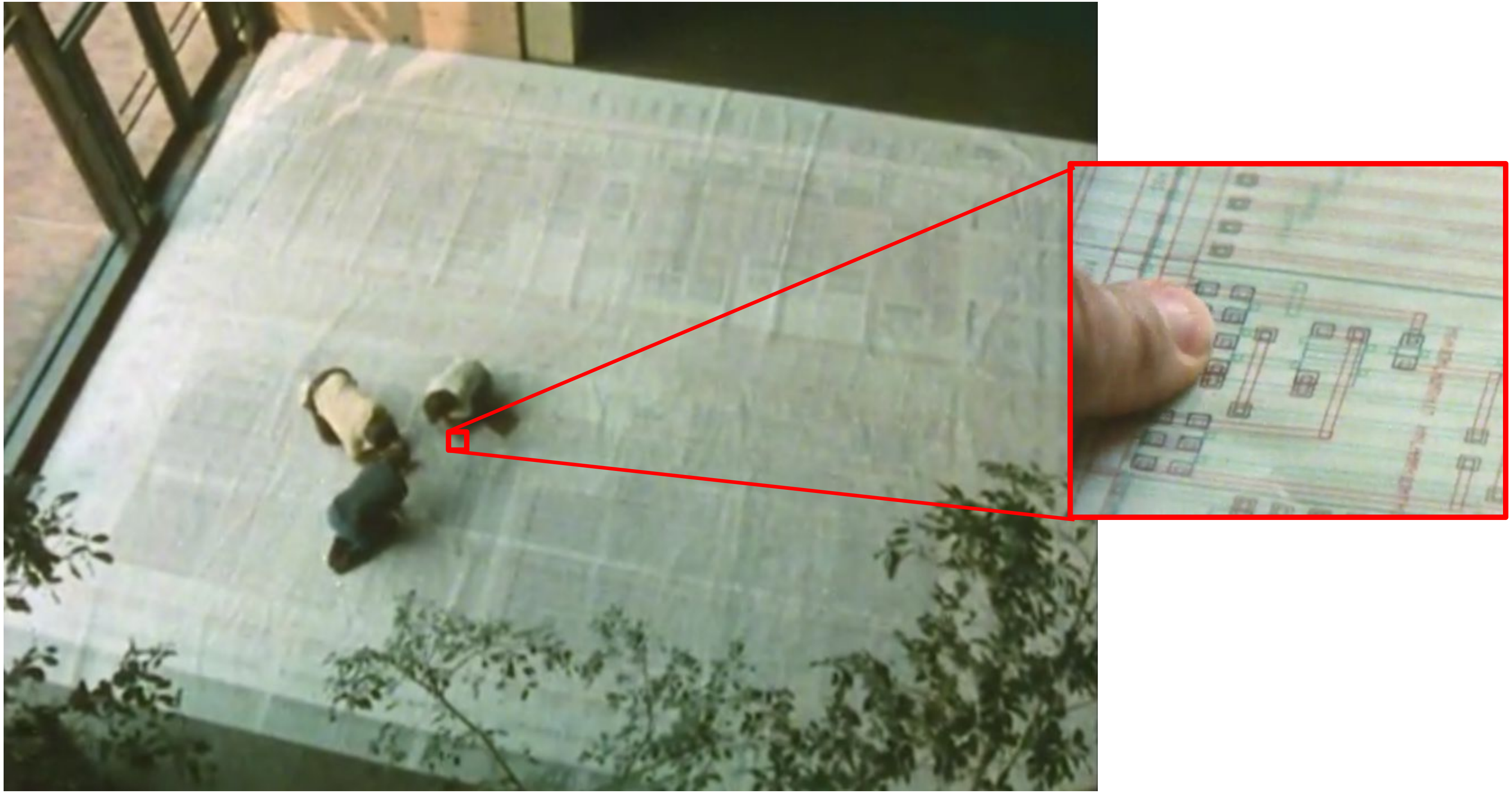
This is 10x fewer qubits than other classically intractable tasks, and lands right in the gap.

Closing remark

The scale of quantum fault tolerance can be daunting...

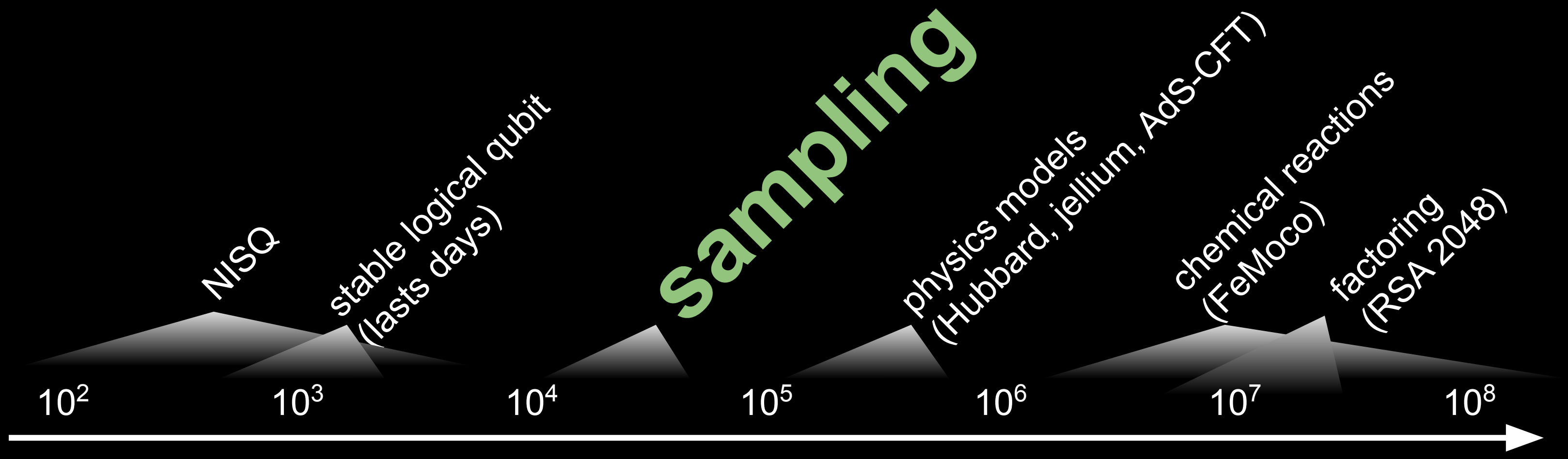


but it's not the first time scale has been daunting



Source: [The Day The Universe Changed (1985) - Episode 1 - It Started with Greeks]

Thanks for listening!



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