Mitigation of readout noise by classical post-processing

based on Quantum Detector Tomography

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POVMs

- POVM (Positive Operator-Valued Measure) **M** with n-outcomes

$$\mathbf{M} = \left(M_1, M_2, \ldots, M_n\right), \qquad \forall i \; M_i \geqslant 0, \qquad \sum_{i=1}^{n} M_i = \mathbb{1}.$$

- **Projective** measurement $\mathbf{P} = \left(P_1, P_2, \ldots, P_n\right)$, with additional requirements

$$\forall_{i,j} P_i P_j = \delta_{i,j} P_i$$

- Born's rule

$$p\left(i|\rho, \mathbf{M}\right) = \mathrm{Tr}\left(\rho M_i\right).$$

Noise model...

- We are interested in 'classical' **noise model**, in which noisy detector **M** is related to ideal detector **P** by **stochastic, invertible map** Λ

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- So such **noise is equivalent to classical post-processing** of statistics!

... and mitigation

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Quantum Detector Tomography

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- **Complexity** – at least 4^n input states... Comment on that at the end of presentation!

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- Let's look at **real data**.

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- **Yesterday around 2PM**, **IBM's qubit** number 2 looked like this:

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$$\underbrace{\mathbf{p}_{\text{exp}}}_{\text{noisy statistics}} = \underbrace{\Lambda\mathbf{p}_{\text{ideal}}}_{\text{"classically noisy" statistics}} + \underbrace{\tilde{\Delta}}_{\text{"coherent disturbance"}} .$$

Non-classical noise

$$\underbrace{\Lambda^{-1}\mathbf{p}_{exp}}_{\text{corrected statistics (?)}} = \underbrace{\mathbf{p}_{\text{ideal}}}_{\text{ideal statistics}} + \underbrace{\Lambda^{-1}\tilde{\Delta}}_{\text{shouldn't be here!}} .$$

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$$D_{TV}(\mathbf{p},\mathbf{q}) := \frac{1}{2}||\mathbf{p}-\mathbf{q}||_1 = \frac{1}{2}\sum_{i=1}^{n}|p_i - q_i| ,$$

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- Now we can **upper bound** interesting distance

$$\underbrace{D_{TV}\left(\Lambda^{-1}\mathbf{p}_{exp},\mathbf{p}_{ideal}\right)}_{\text{How far from ideal statistics?}} \leq \underbrace{||\Lambda^{-1}||_{1\to 1}}_{\text{Norm of 'correction matrix'}} \times \underbrace{D_{op}\left(\mathbf{M},\Lambda\mathbf{P}\right)}_{\text{magnitutude of non-classicity}} .$$

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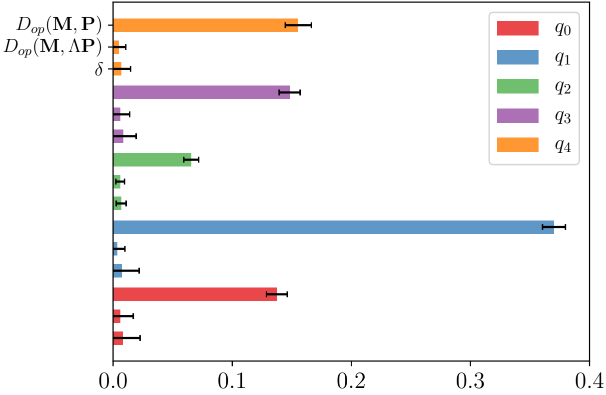

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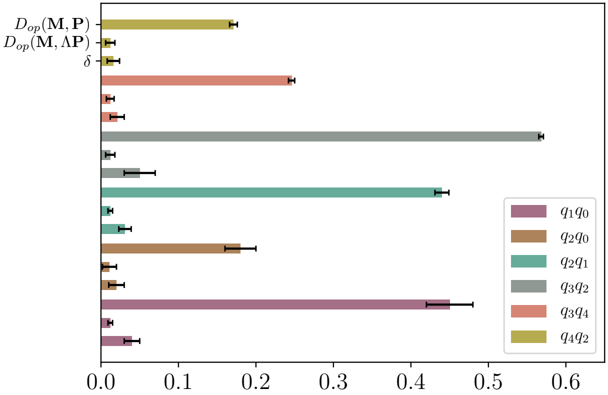

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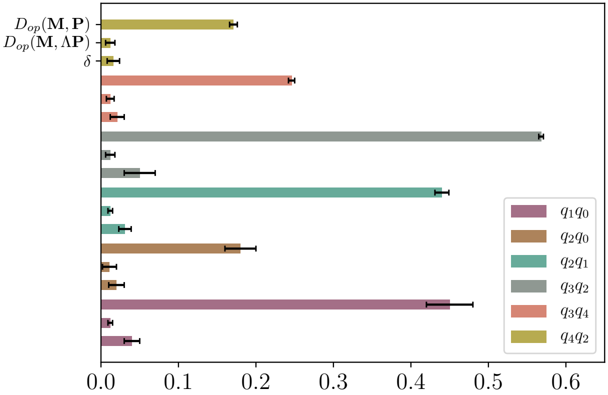

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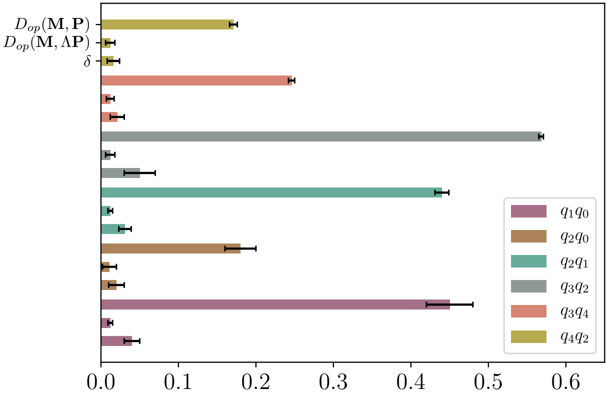

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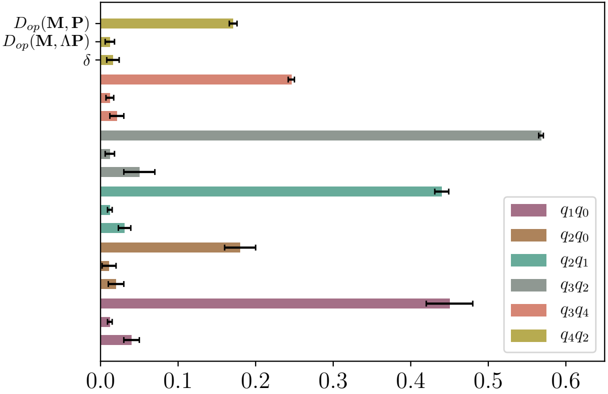

- So it looks **quite good** for our correction... if we have **infinite statistics**. **And what if we do not?**

Finite-size statistics

- In real experiments we **estimate statistics**

$$\underbrace{\mathbf{p}_{exp}}_{\text{theoretical distribution}} \rightarrow \underbrace{\mathbf{p}_{exp}^{est}}_{\textit{frequencies}} .$$

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- If we choose some **acceptable probability** Pr_{wrong}, perform N **runs of experiments** for n-**outcome** measurement, we can get **upper bound**[2]

$$D_{TV}\left(\mathbf{p}_{exp}^{est}, \mathbf{p}_{exp}\right) \leq \sqrt{\frac{\log\left(2^n - 2\right) - \log\left(\Pr_{\text{wrong}}\right)}{2N}} =: \epsilon^* \, .$$

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- Once again we can **compare** it with **non-corrected** scenario

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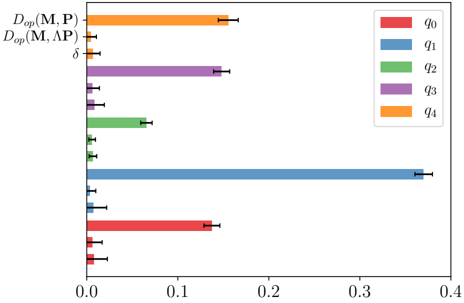
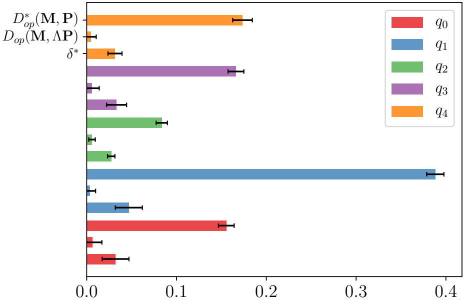
- Let's look at **real data**...

Overall error

(left) $\underbrace{D_{op}(\mathbf{M},\mathbf{P})}_{\text{1. magnitude of readout noise}}$ & $\underbrace{\delta}_{\text{3. error with mitigation}} := \underbrace{\|\Lambda^{-1}\|_{1\to 1}}_{\text{Norm of 'correction matrix'}} \times \underbrace{D_{op}(\mathbf{M},\Lambda\mathbf{P})}_{\text{2. magnitutude of non-classicity}}$

(right) $D^{*}op(\mathbf{M},\mathbf{P}) := D_{op}(\mathbf{M},\mathbf{P}) + \underbrace{\epsilon^{*}}_{\text{statistical errors}}$ & $\delta^{*} := \delta + \underbrace{\|\Lambda^{-1}\|_{1\to 1}\epsilon^{*}}_{\text{propagated statistical errors}}$





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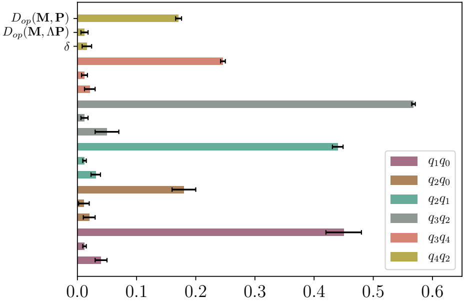

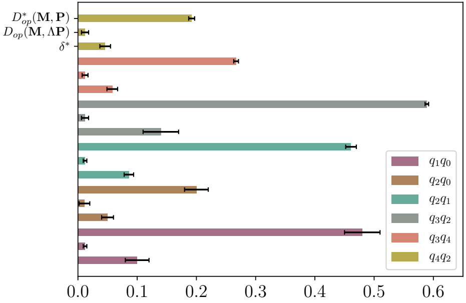

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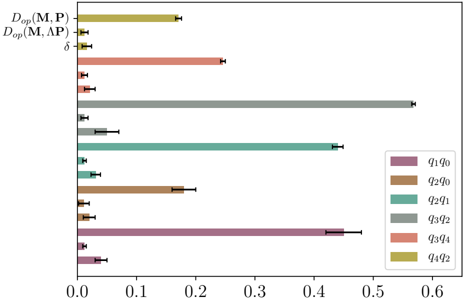

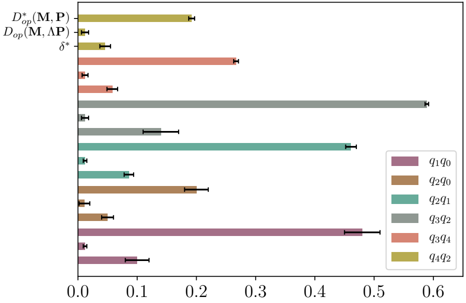

- So our correction **still might work**...

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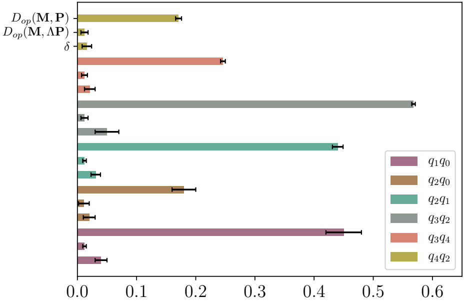

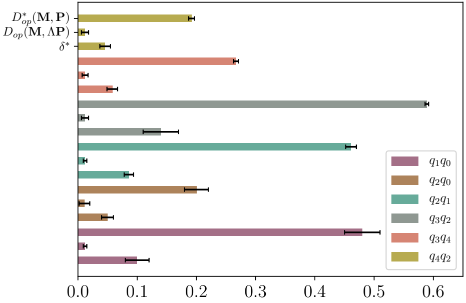

- So our correction **still might work**...
- **Let's check it in practice**...

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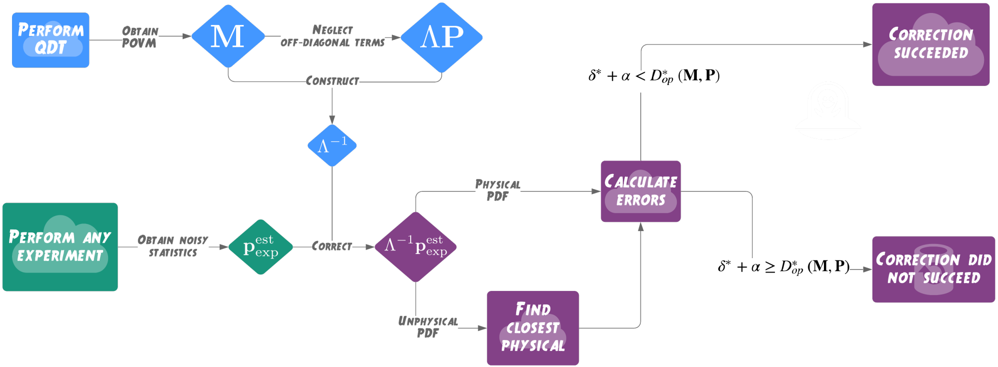
Correction scheme

$$\underbrace{\delta^*}_{\text{errors with mitigation}} := \underbrace{||\Lambda^{-1}||_{1\to 1} D_{op}\left(\mathbf{M}, \Lambda\mathbf{P}\right)}_{\text{coherent errors}} + \underbrace{||\Lambda^{-1}||_{1\to 1}\epsilon^*}_{\text{statistical errors}},$$

$$\underbrace{D^*_{op}\left(\mathbf{M}, \mathbf{P}\right)}_{\text{errors without mitigation}} := \underbrace{D_{op}\left(\mathbf{M}, \mathbf{P}\right)}_{\text{whole readout error}} + \underbrace{\epsilon^*}_{\text{statistical errors}}$$



QST and QPT

- **Single qubit** tasks – Quantum State Tomography and Quantum Process Tomography.

QST and QPT

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- For **two qubits** – Quantum State Tomography.

QST and QPT

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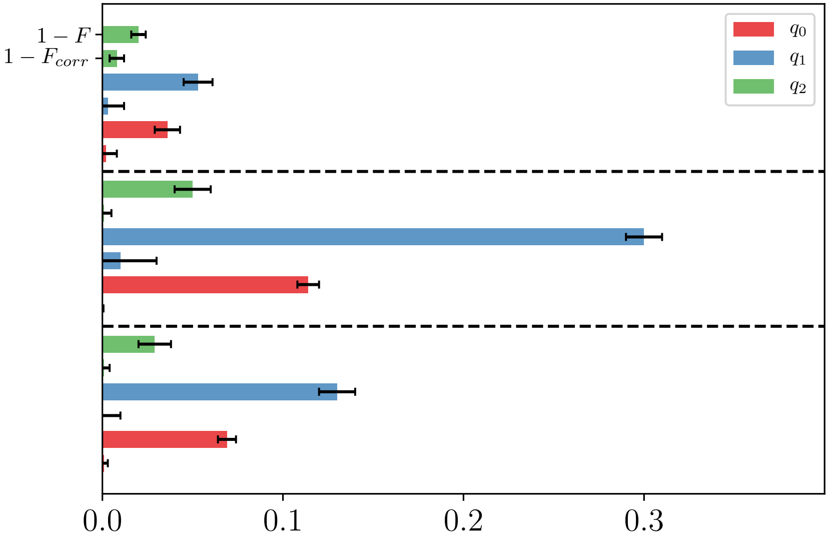
- Figures of merit

$$\underbrace{1-F}_{\text{infidelity without correction}} \quad \& \quad \underbrace{1-F_{corr}}_{\text{infidelity after correction}}$$

Single-qubit QST



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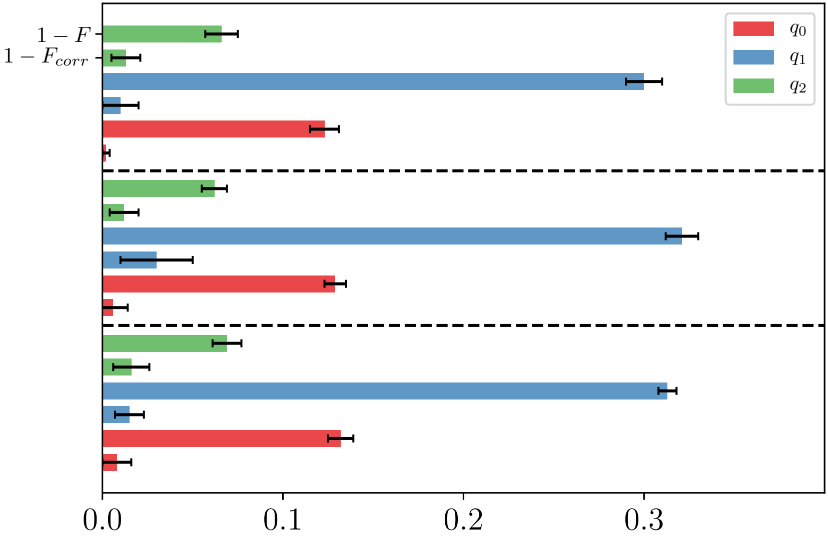
Single-qubit QPT



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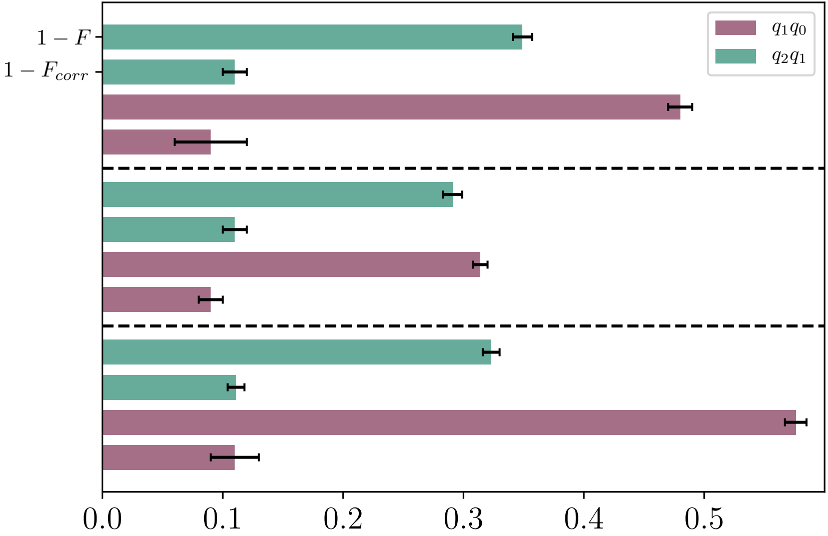
Two-qubit QST



Quantum algorithms

- Quantum algorithms – **Grover's search**[3] (simple oracle) and **Bernstein-Vaziriani**[4] (hidden string).

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[3] L. K. Grover, quant-ph/9605043 (1996),

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Quantum algorithms

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- Figure of merit – **probability of correct outcome** without and with correction.

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Quantum algorithms

Algorithm	Standard	$\underbrace{\mathrm{Corr}(1q \otimes 1q)}_{\text{non-correlated}}$	$\underbrace{\mathrm{Corr}(2q)}_{\text{correlated}}$
Grover's	0.58 ± 0.01	0.70 ± 0.02	0.79 ± 0.02
BV	0.55 ± 0.02	0.63 ± 0.02	0.61 ± 0.02

Five-qubit probablity distributions

- **Five-qubit** task of implementing certain probability distributions.

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$$\underbrace{D_{TV}\left(\mathbf{p}^{est}_{exp}, \mathbf{p}_{ideal}\right)}_{\text{uncorrected PDF}} \qquad \& \qquad \underbrace{D_{TV}\left(\Lambda^{-1}\mathbf{p}^{est}_{exp}, \mathbf{p}_{ideal}\right)}_{\text{corrected PDF}}$$

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Five-qubit probablity distributions

$$\underbrace{\Lambda^{-1}\mathbf{p}_{exp}^{est}}_{\text{might be unphysical}} \rightarrow \underbrace{\mathbf{p}'}_{\text{closest physical}} \qquad \underbrace{\alpha := D_{TV}\left(\Lambda^{-1}\mathbf{p}_{exp}^{est}, \mathbf{p}'\right)}_{\text{upper bound for error introduced by it}}$$

Name	Standard	Corrected	α
Uniform	0.110 ± 0.006	0.100 ± 0.007	0
NOT	0.66 ± 0.02	0 ± 0	0.36 ± 0.09
Mixed	0.196 ± 0.006	0.031 ± 0.008	0.019 ± 0.005

(a) Without accounting for correlations.

Name	Corrected	α
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(b) Accounting for correlations for one pair.

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(b) Accounting for correlations for one pair.

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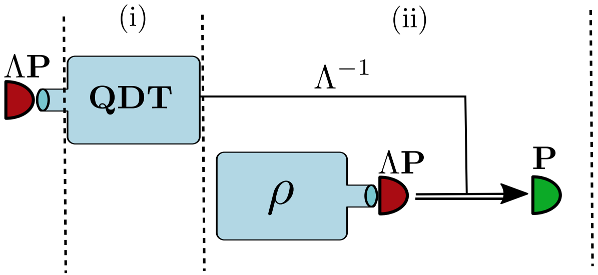
Summary



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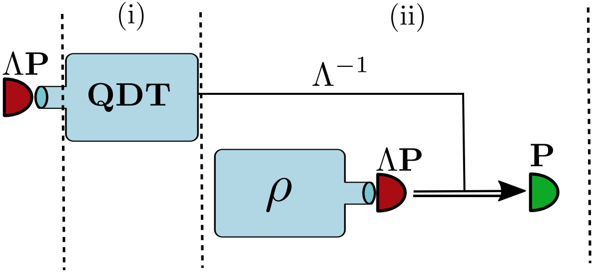
Summary



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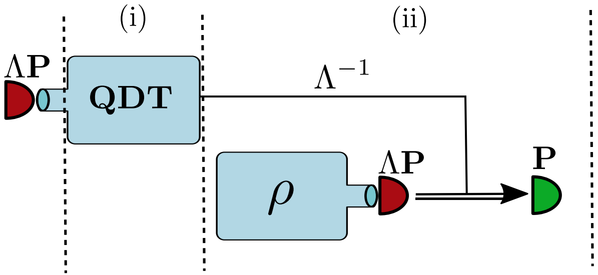

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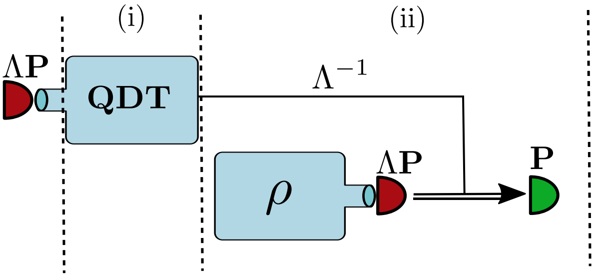

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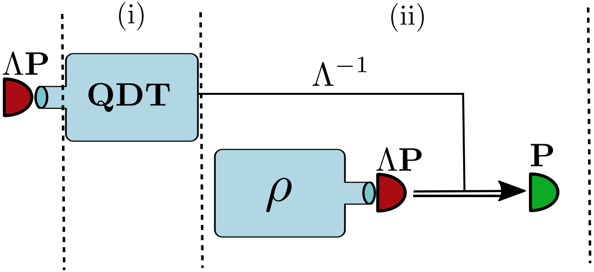

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- **See recent related work:**
  Y. Chen, M. Farahzad, S. Yoo, T.-C. Wei, arXiv:1904.11935 (2018)

Open problems

- **Error bars** for quantum detector tomography[6]...

[6] For state tomography see: M. Guta, J. Kahn, R. Kueng,J. A. Tropp, arxiv:1809.11162 (2018)

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Thank you!

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